Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

SAMPLE SOLUTIONS

Pearson Edexcel Level 3 GCE

Tuesday 20 June 2023

Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced

PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 6 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 7 2 8 2 0 A 0 1 2 0

Pearson

1. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of $3.2\,\text{m}\,\text{s}^{-2}$

Find

(a) the speed of the car after 5 s,

(1)

(b) the distance travelled by the car in the first 5 s.

(2)

a) Because acceleration is constant so you can use SUVAT equations

$v = u + at$

initially at rest ∴ $u = 0\ \text{ms}^{-1}$

$v = 0 + (3.2)(5)$

$= 16\ \text{ms}^{-1}$

b) $s = ut + \frac{1}{2}at^2$

$u = 0$

$s = 0 + \frac{1}{2}(3.2)(5)^2$

$= 40\text{m}$

2.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.4\,\text{m s}^{-2}$

(b) Find the value of F

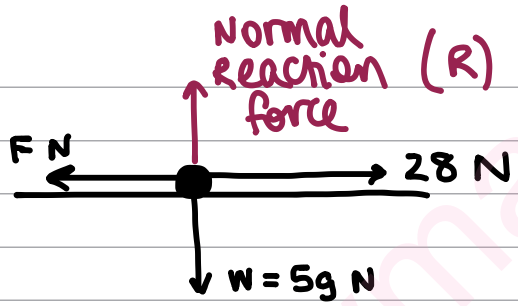
(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

a)



Because the particle isn't accelerating in the vertical direction (↕), the forces must be resolved in the vertical

(i.e. resultant vertical force = 0)

$\therefore\ R = W$

$R = 5g = 5 \times 9.8$

$= 49$ N

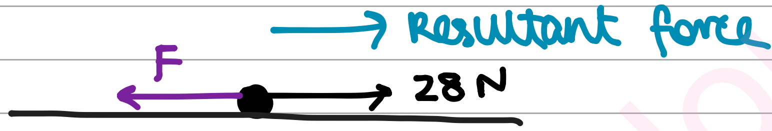
Question 2 continued

b) Because particle has frictional force to left, friction opposes motion, ∴ particle is moving to the right

∴ resultant force is to the right

resultant force → $F = ma$ (mass; ← acceleration)

$F = 5 \times 1.4 = 7$ N



$28 - F =$ resultant force $= 7$ N

$\therefore \quad F = 28 - 7 = 21$ N

c) Friction force (F) $= \mu R$ (coefficient of friction; normal reaction force)

$21 = \mu \times 49$

$\mu = \frac{3}{7} = 0.4286..$

$\therefore \quad \mu = 0.43 \quad (2 s.f.)$

(Total for Question 2 is 4 marks)

3. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$

(3)

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$

(2)

(c) the acceleration of P at time t seconds

(2)

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$

(2)

a) $v = \begin{pmatrix} t^2 - 3t + 7 \\ 2t^2 - 3 \end{pmatrix}$

when $t=0$ $\quad v = \begin{pmatrix} 0 - 0 + 7 \\ 0 - 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$

speed is the magnitude of velocity

speed $= \sqrt{7^2 + (-3)^2} = \sqrt{49 + 9} = \sqrt{58}$ $\underline{\underline{\mathrm{ms^{-1}}}}$

don't forget units

b) when P is moving parallel to $(i+j)$,

it means that the velocity (i.e. motion) is parallel to the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$v = \begin{pmatrix} t^2 - 3t + 7 \\ 2t^2 - 3 \end{pmatrix} \uparrow\uparrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

parallel means that the vectors are multiples of each other

Question 3 continued

$$\therefore \quad k\begin{pmatrix} t^2 - 3t + 7 \\ 2t^2 - 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\cancel{k}(t^2 - 3t + 7) = 1 = \cancel{k}(2t^2 - 3)$$

$$t^2 - 3t + 7 = 2t^2 - 3$$

$$t^2 + 3t - 10 = 0$$

← factorise

$$(t+5)(t-2) = 0$$

$$t = \cancel{-5} \cup 2\text{ s}$$

↑ but t cannot be negative

$$\therefore \quad t = 2\text{ s}$$

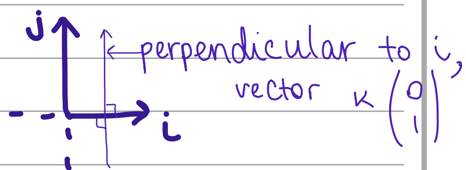
c) acceleration is the derivative of velocity

$$v = \begin{pmatrix} t^2 - 3t + 7 \\ 2t^2 - 3 \end{pmatrix} \qquad a = \frac{d}{dt}(v) = v'$$

$$v' = a = \begin{pmatrix} 2t - 3 \\ 4t \end{pmatrix} \qquad a = (2t - 3)i + (4t)j$$

↑ same format as qu.

d) when acceleration is perpendicular to i, parallel to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$$k\begin{pmatrix} 2t - 3 \\ 4t \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad k(2t - 3) = 0 \qquad k(4t) = 1$$

$$\therefore \quad 2t - 3 = 0 \qquad \text{but } k \neq 0$$

$$t = \frac{3}{2}\text{ s}$$

(Total for Question 3 is 9 marks)

4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P passes through the point A.

At time $t = 5$ s, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})$ m.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})$ m.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

a) velocity is the integral of acceleration

$$v = \int a \, dt$$

$$a = \begin{pmatrix} 2.4 \\ 1 \end{pmatrix} \qquad v = \begin{pmatrix} 2.4t + c \\ t + d \end{pmatrix}$$

different constants of integration

when passes P :

$t = 0$

$$v = \begin{pmatrix} -16 \\ -3 \end{pmatrix} = v = \begin{pmatrix} 2.4(0) + c \\ (0) + d \end{pmatrix}$$

$$\begin{pmatrix} -16 \\ -3 \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$\therefore v = \begin{pmatrix} 2.4t - 16 \\ t - 3 \end{pmatrix}$$

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Question 4 continued

when passes B: $t = 5$

$$v = \begin{pmatrix} 2.4(5) - 16 \\ (5) - 3 \end{pmatrix} = \begin{pmatrix} 12 - 16 \\ 5 - 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

speed is magnitude of velocity

$\sqrt{4 \times 5} = \sqrt{2^2 \times 5}$

$$\text{speed} = \sqrt{(-4)^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ ms}^{-1}$$

b) METHOD 1: (integrate velocity)

displacement is the integral of velocity

displacement $\rightarrow$ $s = \int v \, dt$

$$v = \begin{pmatrix} 2.4t - 16 \\ t - 3 \end{pmatrix} \qquad s = \int \begin{pmatrix} 2.4t - 16 \\ t - 3 \end{pmatrix} dt$$

$$= \begin{pmatrix} 1.2t^2 - 16t + a \\ 0.5t^2 - 3t + b \end{pmatrix}$$

at A: $t = 0$

$$s = \begin{pmatrix} 44 \\ -10 \end{pmatrix}$$

$$s = \begin{pmatrix} 44 \\ -10 \end{pmatrix} = \begin{pmatrix} 1.2(0)^2 - 16(0) + a \\ 0.5(0)^2 - 3(0) + b \end{pmatrix}$$

$$\begin{pmatrix} 44 \\ -10 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \quad \therefore \quad s = \begin{pmatrix} 1.2t^2 - 16t + 44 \\ 0.5t^2 - 3t - 10 \end{pmatrix}$$

Question 4 continued

METHOD 2 : (use SUVAT vector equations)

$s = \bar{v}_A t + \frac{1}{2}\bar{a}t^2 + \bar{s}_A$

$$s = \begin{pmatrix} -16 \\ -3 \end{pmatrix} t + \frac{t^2}{2}\begin{pmatrix} 2.4 \\ 1 \end{pmatrix} + \begin{pmatrix} 44 \\ -10 \end{pmatrix}$$

$$s = \begin{pmatrix} 1.2t^2 - 16t + 44 \\ 0.5t^2 - 3t - 10 \end{pmatrix}$$

when $t = T$, passes through C

$$s = \begin{pmatrix} 4 \\ c \end{pmatrix} = \begin{pmatrix} 1.2T^2 - 16T + 44 \\ 0.5T^2 - 3T - 10 \end{pmatrix}$$

$1.2T^2 - 16T + 40 = 0 \leftarrow$ quadratic solver

$T = 10 \cup \cancel{\frac{10}{3}}$ but $T > 5$

$\therefore T = 10 \text{ s}$

c) $c = 0.5(10)^2 - 3(10) - 10$

$= 0.5(100) - 30 - 10$

$= 50 - 40$

$= 10$

5.

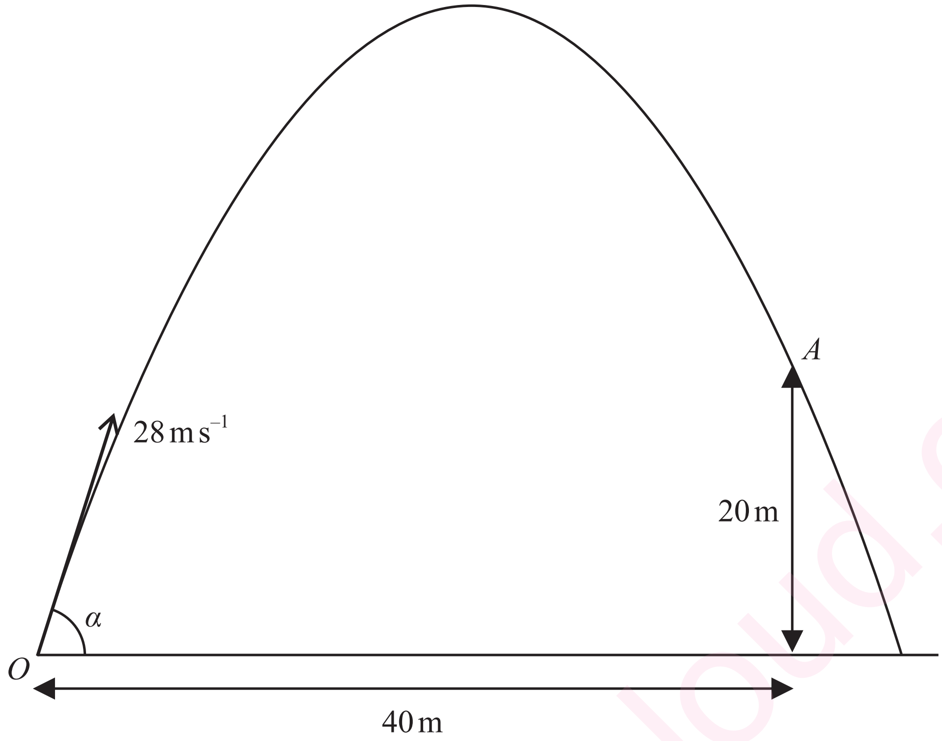


Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

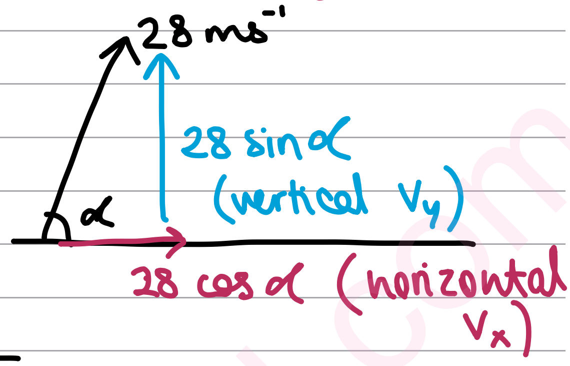
The model does not include air resistance.

(d) State one other limitation of the model.

(1)

Question 5 continued

a) Horizontal velocity is constant (assuming no air resistance)



$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

time at A

$$T = \frac{\text{horizontal distance to A}}{\text{horizontal component velocity}}$$

$$T = \frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha}$$

b) In the vertical direction, there is only 1 force (gravity), acting downwards, so there is constant acceleration & can use SUVAT equations

(taking upwards direction to be positive)

$$s = ut + \frac{1}{2}at^2$$

$$20 = (28\sin\alpha)T - \frac{g}{2}T^2$$

(gravity acts down, so negative)

$$T = \frac{10}{7\cos\alpha}$$

$$20 = \frac{\cancel{28}^{4}\sin\alpha \times 10}{\cancel{7}\cos\alpha} - \frac{g}{2}\left(\frac{10}{7\cos\alpha}\right)^2$$

$$20 = 40\tan\alpha - \frac{9.8 \times 100}{2 \times 49\cos^2\alpha}$$

$$20 = 40\tan\alpha - 10\sec^2\alpha$$

$$2 = 4\tan\alpha - \sec^2\alpha$$

Question 5 continued

$$\cos^2\theta + \sin^2\theta = 1 \qquad (\div \text{ by } \cos^2\theta)$$

$$1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$2 = 4\tan\alpha - \sec^2\alpha$$

$$2 = 4\tan\alpha - (1+\tan^2\alpha)$$

← replace by identity

$$3 = 4\tan\alpha - \tan^2\alpha$$

$$\tan^2\alpha - 4\tan\alpha + 3 = 0$$

c) solve quadratic equation with $\tan\alpha$

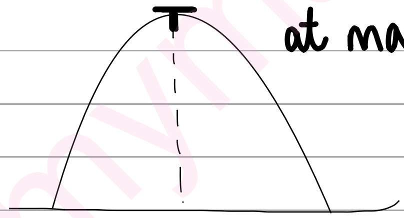
$\tan\alpha = m$

$$m^2 - 4m + 3 = 0$$

$$(m-3)(m-1) = 0$$

$m = \tan\alpha = 1 \cup 3$

Max height :



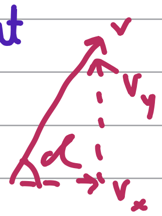
at max height : $v_y = 0$

vertical component of velocity

SUVAT : $v^2 = u^2 + 2as$

max height displacement

$$v^2 = 0 = (v_{y\,(\text{original})})^2 + 2as$$



← using pythagoras

$v_y = v\sin\alpha$

$$0 = (v\sin\alpha)^2 - 2gs$$

$$s = \frac{v^2\sin^2\alpha}{2g}$$

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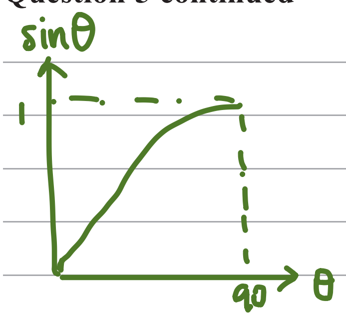
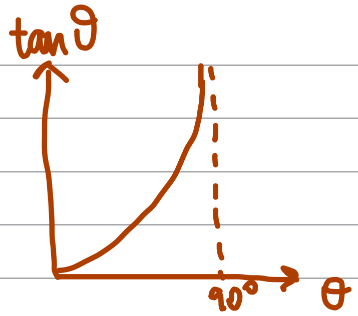
Question 5 continued

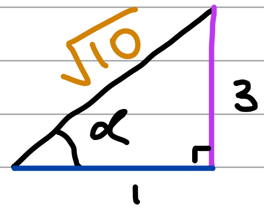




The bigger the value of $\tan\theta$
the bigger the value of $\sin\theta$

$\therefore$ for the highest value of, choose the largest value of $\sin\theta$ & $\therefore$ the larger value of $\tan\theta$

$\tan\alpha = 1 \cup 3 \quad \therefore$ larger value : $\tan\alpha = 3$



$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} = 3$

using pythagoras, hypotenuse $= \sqrt{3^2+1^2} = \sqrt{10}$

$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{\sqrt{10}}$

max height $= \frac{v^2\sin^2\alpha}{2g} = 28^2 \times \left(\frac{3}{\sqrt{10}}\right)^2 \times \frac{1}{2\times 9.8}$

$= \frac{784 \times 9}{2\times 9.8 \times 10} = 36$ m

d) → dimensions of ball (not a particle)

→ there could be wind, affecting the ball's velocity (not in freefall with only gravity acting)

(Total for Question 5 is 11 marks)

6.

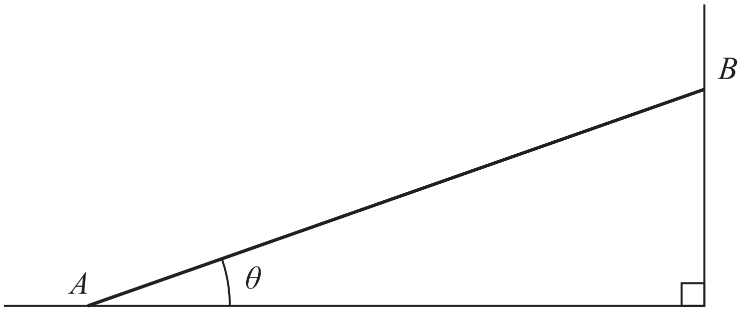


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

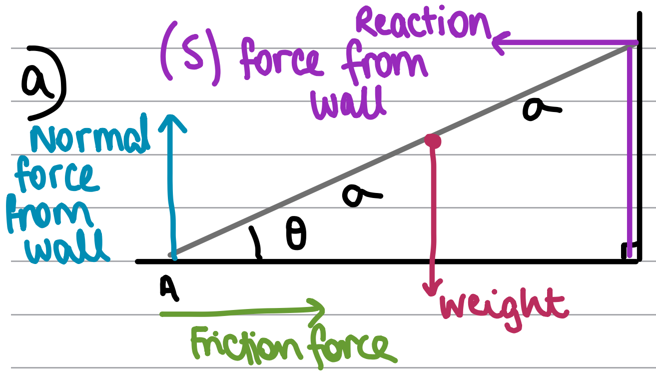
A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

Question 6 continued

a)



For ruler to be in equilibrium, both vertical & horizontal forces must be balanced

The weight balances the normal force from the wall

The reaction force from the wall to the left must be balanced by a friction force to the right

b) Taking moments about A:

Anticlockwise moment: $S \times$ perpendicular distance from force to pivot

$S \times 2a\sin\theta$

Clockwise moment: $W \times$ perpendicular distance from force to pivot

$W \times a\cos\theta$

For the ruler to be in equilibrium:

anticlockwise moment = clockwise moment

$$S \times 2a\sin\theta = W \times a\cos\theta$$

$$S = W \times \frac{\cancel{a}\cos\theta}{2\cancel{a}\sin\theta} = \frac{Mg}{2\tan\theta} = \frac{1}{2}Mg\cot\theta$$

Question 6 continued

c) Friction force = Normal contact force from wall $\times$ μ ← coefficient of friction

Normal contact force from wall = weight (resolve forces vertically)

Friction force = S

$$\mu M g = \frac{1}{2} M g \cot\theta$$

$$\mu = \frac{\cancel{Mg}\cot\theta}{2\times\cancel{Mg}}$$

$\tan\theta = \frac{3}{4}$ $\quad \cot\theta = \frac{4}{3}$

$$\mu = \frac{1}{2}\times\frac{4}{3} = \frac{2}{3}$$

d) magnitude of resultant force at A

$$= \sqrt{(\text{normal})^2 + Fr^2}$$

$$\sqrt{(Mg)^2 + \left(\frac{2}{3}Mg\right)^2}$$

$$= \sqrt{\frac{13}{9}(Mg)^2}$$

$$= \frac{\sqrt{13}\,Mg}{3}$$

e) If the centre of mass is closer B, then the clockwise moment would be bigger. This means that the anticlockwise moment must be bigger, so S must also be larger.